The Inferential Test is Always True. Think of it as a Law.

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The Inferential Test for Expert Testimony is a theorem--something that can be proven through deductive logic. I will do so now by translating the Test into a symbolic statement using logical operator notation and then testing that statement with a truth table and a proof.

As I do so, I refer the reader to a recently published logic textbook for further information.[1] This textbook is also available for use for 180 days via an internet purchase.[2] I have included page numbers from this textbook in brackets following several of the concepts; this will allow the reader to verify the information and to do further reading.

The **Inferential Test for Expert Testimony** is:

> **One can be reasonably certain if witness accounts of the past are consistent or not consistent with physical evidence in the present, but one cannot reliably surmise past events from physical evidence unless there is only one plausible explanation for that evidence.**

Each portion of this statement will now be represented by a variable, consisting of a capital letter. "A" represents the first part of the sentence before the "but." "B" represents the second part of the sentence after the "but." "C" represents being certain. "E" represents the exception at the end (unless...). The Test is now:

1. C if A, but not C if B unless E

"Not C" is a negation of C, meaning "not certain."

In logical operator notation, this is:

2. $(A \rightarrow C) \bullet [(B \rightarrow \sim C) \vee E]$

A *conditional statement*--also known as a "conditional"--is a statement in the form of "If..., then..." [pp. 17- 20]. "C if A" is a *stylistic variant* of "If A, then C," and "C if B" is a stylistic variant of "If B, then C" [p. 18]. The operator for a conditional is an arrow. "Not C" is a *negation* of C [p. 20], and the operator for this is a tilde. The operator for a *disjunction* is a vee. A disjunction is a statement in the form, "Either A or B" [p. 20]. The word, "unless," is a stylistic variant of "or" [p. 283]. The operator for a *conjunction* is a dot. A conjunction is a statement in the form, "A and B," and the word, "but," is a stylistic variant of "and" [p. 282].

Parentheses and brackets in logical operator notation indicate the functions that are performed first. The parentheses nested within the brackets mean that the operations in the parentheses are done before the operations in the brackets. The internally nested functions are done first, and the other functions in sequence are done as each set of outward parentheses and brackets is encountered. The function in the middle using the dot is not enclosed in parentheses or brackets, and this is done last [pp. 288 - 298]. It is the *major logical operator* that governs the entire statement [pp. 280 - 281].

A person does not have to be certain, even though he or she can be certain. This is symbolized by the statement, C v ~C. This statement is a *tautology*, or a necessary truth [p. 332]. It is always true that one is either certain or not certain [p. 373]:

> 3. [A → **(C v ~C)**] • [(B → ~C) v E] (Bold type indicates new substitution from previous formula)

But if one chooses to be certain, he or she can do so and the statement is still valid. This is demonstrated using a *conjunction of premises* and a *corresponding conditional* [p. 413]. Using a conjunction of premises and a corresponding conditional is a way of putting a valid argument form into a conditional statement. For example, the valid argument form, *modus ponens* (If *p*, then *q*; *p*; therefore *q*) [p. 15] can be stated in operator notation as [(*p* → *q*) • *p*] → *q.* This is now done below, indicating the choice to be certain rather than not certain:

> 4. [([A → (C v ~C)] **● C) → (A → C)**] • [(B → ~C) v E]

Now on to further substitutions (see #5 below). The first part of the inferential test (A) utilizes "witness accounts of the past" and "physical evidence in the present." P represents the witness accounts and Q represents the physical evidence. I choose to use these letters for these items because I am accustomed to writing scientific conditional statements using the letters P and Q for *"antecedent"* and *"consequent"*--the two portions of a conditional statement [p. 17]. Since the antecedent in a forensic analysis comes before the consequent in time, P will represent the "witness accounts of the past" and Q the consequent "physical evidence in the present."

The term, "consistent or not consistent with," does not refer to logical consistency or inconsistency as described in the text [pp. 335 - 336]. The first part of the inferential test applies either *modus ponens* or *modus tollens* to the analysis. In logical operator notation, *modus ponens* is [(*p* → *q*) • *p*] → *q* and *modus tollens* is [(*p* → *q*) • ~*q*] → ~*p* [p. 361]. Using *simplification* [p. 361], both are simplified to *p* → *q* and ~*q* → ~*p*, respectively. Both are logically equivalent to each other [p. 334] and expressed in the Inferential Test formula as the usual past event/present evidence conditional and its *contrapositive* [p. 372]:

5. [(**[[(P → Q) v (~Q → ~P)]** → (C v ~C)] • C) → [(**[(P → Q) v (~Q → ~P)]** → C)] • [(B → ~C) v E]

What about B? B is put in the typical conditional for surmising past events from physical evidence:

6. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → [([(P → Q) v (~Q → ~P)] → C)] • ([**(Q → P)** → ~C] v E)

Now we come to E. We can be certain that physical evidence can be used to surmise past events if there is only one plausible explanation for the evidence. Hence, the biconditional:

7. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → [([(P → Q) v (~Q → ~P)] → C)] • ([(Q → P) → ~C] v **[(Q ↔ P) → C]**)

A *biconditional statement*, symbolized by a double arrow, is another way of stating, "If *p*, then *q*; and if *q*, then *p*": (*p* → *q*) • (*q* → *p*) [p. 383]. It represents the term, *if and only if* [p. 18], allowing one to infer past events from physical evidence as long as there is only one plausible explanation for that evidence. A biconditional using a simplification of (*p* → *q*) • (*q* → *p*) can truthfully state *q* → *p* in that instance.

Using the exception, a person does not have to be certain, even though he can be certain:

8. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → [([(P → Q) v (~Q → ~P)] → C)] • ([(Q → P) → ~C] v ([(Q ↔ P) → **(C v ~C)**])

But, once again, if he or she chooses to be certain, he or she can do so and the statement is still valid. Once again, using a conjunction of premises and a corresponding conditional:

9. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → ([(P → Q) v (~Q → ~P)] → C)] • [[(Q → P) → ~C] v (([(Q ↔ P) → (C v ~C)]) **• C) → [(Q ↔ P) → C])**]

Examine the final logical operator statement for the Inferential Test:

10. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C)] → ([(P → Q) v (~Q → ~P)] → C)] • [[(Q → P) → ~C] v (([(Q ↔ P) → (C v ~C)]) • C) → [(Q ↔ P) → C])]

Notice once again that the statement is a series of minor statements nested in parentheses, indicating which functions are to be evaluated first. This statement in logical operator notation allows for testing with a *truth table* [pp. 302 - 309]. I used the truth table below to test the validity of a large class of arguments.

Truth Table for the Inferential Test

P	Q	C	[[[[(P → Q)	v	(~Q → ~P)]	→	(C v ~C)]
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

• C)]	→	([[(P → Q)	v	(~Q → ~P)]	→ C)]
T	T	T	T	T	T
F	T	T	T	T	F
T	T	F	F	F	T
F	T	F	F	F	T
T	T	T	T	T	T
F	T	T	T	T	F
T	T	T	T	T	T
F	T	T	T	T	T

•	[[(Q → P)	→	~C]	v	((([(Q ↔ P)
T	T	F	F	T	T
T	T	T	T	T	T
T	T	F	F	T	F
T	T	T	T	T	F
T	F	T	F	T	F
T	F	T	T	T	F
T	T	F	F	T	T
T	T	T	T	T	T

→	(C v ~C)])	• C)	→	[(Q ↔ P)	→ C])]
T	T	T	T	T	T
T	T	F	T	T	F
T	T	T	T	F	T
T	T	F	T	F	T
T	T	T	T	F	T
T	T	F	T	F	T
T	T	T	T	T	T
T	T	F	T	T	F

For each *atomic statement* P, Q and C [p. 278], varying combinations of true (T) and false (F) are entered into the first three columns of the table so that all possible combinations are tested. Then, each operation is tested in the order directed by the nested parentheses. For each operator--the tilde, the dot, the vee, the arrow and the double-arrow--varying combinations of T and F can be ascertained as follows:

For negations (tilde), each time a T is assigned to an atomic statement, an F is assigned to the negation, and each time an F is assigned to an atomic statement, a T is assigned to the negation [p. 303].

For conjunctions (dot), T is assigned if both atomic statements are true; otherwise, F is assigned [p. 303 - 304].

For disjunctions (vee), F is assigned if both atomic statements are false; otherwise, T is assigned [p. 304].

For conditionals (arrow), F is assigned if the consequent is false and the antecedent is true; otherwise, T is assigned [pp. 304 - 306].

For biconditionals (double-arrow), T is assigned when the atomic statements have the same truth assignment (T and T, F and F) and F when they have differing truth assignments (T and F, F and T) [pp. 306 - 308].

To simplify an already complex table, T's are assigned to all the boxes in the column for the tautology, C v ~C. Also, the same truth assignments are used for P → Q and its contrapositive, ~Q → ~P; and for each operation using the single variable, C, I placed the operator in the same box as C in order not to recopy the values for C.

Note the column enclosed by the rectangle. The rectangle represents the conjunction of the two parts of the inferential test--the major logical operator. This operation is true for every assignment of values in the truth table. Hence, the column enclosed in the rectangle indicates a tautology. Tautologies of statement logic are identical to theorems of statement logic [page 411]. As such--as complex as this entire statement is in symbolic logic--***the inferential test is a theorem. It is a necessary truth and cannot be false under any possible circumstances.***

Another way to demonstrate that the Inferential Test is a necessary truth is through a proof [p. 346]. Refer to pp. 345 - 398 and pp. 411 - 415 for information on the construction of proofs.

Proof for the Inferential Test

1. P → Q	Assume (for CP)
2. ~Q → ~P	1, Cont
3. (P → Q) v (~Q → ~P)	1, 2, Add
4. C v ~C	LEM
5. [(P → Q) v (~Q → ~P)] → (C v ~C)	1 - 4, CP
6. C	Assume (for CP)
7. ~~C	6, DN
8. [[(P → Q) v (~Q → ~P) → (C v ~C)] • C	5, 6, Conj
9. [(P → Q) v (~Q → ~P)] → C	5, 7, DS
10. ([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → ([(P → Q) v (~Q → ~P)] → C)	6 - 9, CP
11. Q ↔ P	Assume (for CP)
12. (Q ↔ P) → (C v ~C)	11, 4, CP
13. [(Q ↔ P) → (C v ~C)] • C	6, 12, Conj
14. (Q ↔ P) → C	12, 7, DS
15. ([(Q ↔ P) → (C v ~C)] • C) → [(Q ↔ P) → C]	11 - 14, CP
16. [(Q → P) → ~C] v ([(Q ↔ P) → (C v ~C)] • C) → [(Q ↔ P) → C])	15, Add
17. [([[(P → Q) v (~Q → ~P)] → (C v ~C)] • C) → ([(P → Q) v (~Q → ~P)] → C)] • ([[(Q → P) → ~C] v ([(Q ↔ P) → (C v ~C)] • C) → [(Q ↔ P) → C])]	9, 16, Conj
CP → Conditional Proof; Cont → Contraposition; Add → Addition; LEM → Law of the Excluded Middle; DN → Double Negation; DS → Disjunctive Syllogism; Conj → Conjunction.	

One nearly final note. If a scientist surmises past events from physical evidence and then claims certainty, then the formula would look like this:

> 11. ([(P → Q) v (~Q → ~P)] → (C v ~C) • C) → [(P → Q) v (~Q → ~P) → C] • **[([(Q → P) → (C v ~C)] • C) → [(Q → P) → C]]** (The bold portion indicates being certain while inferring from physical evidence to past events).

A truth table completed for this statement would also indicate a tautology. Why is this?

It is always true that any scientist can choose to be certain about anything, but he or she might be *certainly incorrect* to do so.

Note the additional truth tables for *modus ponens*, *modus tollens*, and *affirming the consequent*.

Modus Ponens

P	Q	P → Q,	P,	∴ Q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Modus Tollens

P	Q	P → Q,	~Q,	∴ ~P
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Affirming the Consequent

P	Q	$P \rightarrow Q$,	Q,	$\therefore$ P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

If one of the conclusions in a truth table is false when all the premises are true, then the argument is demonstrated to be invalid [p. 312]. The only truth table where this is the case is the one for *affirming the consequent* (note the red rectangle), demonstrating it to be an invalid argument form. Any argument in this form is unsound since the conclusion cannot be guaranteed to be truthful if all the premises are truthful [p. 8].

This is why the word, "reasonably," is inserted into the Inferential Test. Reasonable scientists who understand what they are doing will not offer unsound arguments with certainty. They will state that they are uncertain whenever they reason from physical evidence to past events--i.e., $[(Q \rightarrow P) \rightarrow {\sim}C]$. Otherwise, great harm may be done. The formula in #11 demonstrates that the potential for a scientist to perpetrate great harm is also a necessary truth.

Another nearly final note. In real life situations, biconditional statements such as the one applied in the exception are rarely true. This is why the exception should be applied with great caution. Great caution is appropriate because it takes only *one other plausible explanation* for physical evidence to falsify the use of the biconditional exception. This is demonstrated through *modus tollens*:

1. If a scientist or a jury uses the biconditional exception, then they claim that no other plausible explanation for the evidence exists ($P \rightarrow Q$, with P representing "a scientist or a jury uses the biconditional exception" and Q representing "no other plausible explanation for the evidence exists").
2. Another plausible explanation for the evidence exists (${\sim}Q$).
3. Therefore, a scientist or a jury cannot use the biconditional exception ($\therefore {\sim}P$).

No scientist testifying in court should be allowed to apply the biconditional exception to past events considered by a jury. This exception is not a scientific principle but a way for a jury—after hearing all available arguments and evidence—to make a decision "beyond a reasonable doubt" in a circumstantial evidence case. A death investigator considering a manner of death with only circumstantial evidence is wise to remain tentative and uncertain, always remaining open to other possibilities.

Once more, the **Inferential Test for Expert Testimony**:

> **One can be reasonably certain if witness accounts of the past are consistent or not consistent with physical evidence in the present, but one cannot reliably surmise past events from physical evidence unless there is only one plausible explanation for that evidence.**

Recognize this as true under all circumstances because I have now proven it. It is a theorem--a necessary truth. Hopefully other scientists and the courts will figure out its necessity.

Addendum

On further consideration, I realized that someone someday might ask me the following questions: The Inferential Test may be true in a particular case under particular circumstances but what evidence do I have that it is true for *all persons and situations*? What evidence do I have of its universality?

So I decided to apply a proof using *predicate logic* [pp. 419-501].

Proof for the Inferential Test Using Predicate Logic

1. Pdc → Qsc	Assume (for CP)
2. ~Qsc → ~Pdc	1, Cont
3. (Pdc → Qsc) v (~Qsc → ~Pdc)	1, 2, Add
4. Csc v ~Csc	LEM
5. [(Pdc → Qsc) v (~Qsc → ~Pdc)] → (Csc v ~Csc)	1 - 4, CP
6. Csc	Assume (for CP)
7. ~~Csc	6, DN
8. [[(Pdc → Qsc) v (~Qsc → ~Pdc) → (Csc v ~Csc)] • Csc	5, 6, Conj
9. [(Pdc → Qsc) v (~Qsc → ~Pdc)] → Csc	5, 7, DS
10. ([[(Pdc → Qsc) v (~Qsc → ~Pdc)] → (Csc v ~Csc)] • Csc) → ([(Pdc → Qsc) v (~Qsc → ~Pdc)] → Csc)	6 - 9, CP
11. Qsc ↔ Pc	Assume (for CP)
12. (Qsc ↔ Pc) → (Csc v ~Csc)	11, 4, CP
13. [(Qsc ↔ Pc) → (Csc v ~Csc)] • Csc	6, 12, Conj
14. (Qsc ↔ Pc) → Csc	12, 7, DS
15. ([(Qsc ↔ Pc) → (Csc v ~Csc)] • Csc) → [(Qsc ↔ Pc) → Csc]	11 - 14, CP
16. [(Qsc → Pc) → ~Csc] v ([(Qsc ↔ Pc) → (Csc v ~Csc)] • Csc) → [(Qsc ↔ Pc) → Csc])	15, Add
17. [([[(Pdc → Qsc) v (~Qsc → ~Pdc)] → (Csc v ~Csc)] • Csc) → ([(Pdc → Qsc) v (~Qsc → ~Pdc)] → Csc)] • ([[(Qsc → Pc) → ~Csc] v ([(Qsc ↔ Pc) → (Csc v ~Csc)] • Csc) → [(Qsc ↔ Pc) → Csc])]	9, 16, Conj
18. (x)(y)(z)([([[(Pxz → Qyz) v (~Qyz → ~Pxz)] → (Cyz v ~Cyz)] • Cyz) → ([(Pxz → Qyz) v (~Qyz → ~Pxz)] → Cyz)] • ([[(Qyz → Pz) → ~Cyz] v ([(Qyz ↔ Pz) → (Cyz v ~Cyz)] • Cyz) → [(Qyz ↔ Pz) → Cyz])])	17, UG
CP → Conditional Proof; Cont → Contraposition; Add → Addition; LEM → Law of the Excluded Middle; DN → Double Negation; DS → Disjunctive Syllogism; Conj → Conjunction; UG → Universal Generalization	
d → A particular defendant, witness, or witnesses; s → A particular scientist; c → A particular case.	
Pdc → Antecedent past events (P) as witnessed by a particular defendant, witness or witnesses (d) in a particular case (c).	
Qsc → Consequent physical evidence (Q) as observed by a particular scientist (s) in a particular case (c).	
Csc → Reasonable certainty (C) as expressed by a particular scientist (s) in a particular case (c).	
Pc → Antecedent past events (P) in a particular case (c).	
(x)(y)(z)Pxz → For all x, y, and z, antecedent past events (P) are witnessed by x in z.	
(x)(y)(z)Qyz → For all x, y, and z, consequent physical evidence (Q) is observed by y in z.	
(x)(y)(z)Cyz → For all x, y, and z, reasonable certainty (C) is expressed by y in z.	
(x)(y)(z)Pz → For all x, y, and z, antecedent past events (P) are in z.	

The final step in this proof uses an inferential rule known as **universal generalization**. This allows generalization of the Inferential Test to all persons and situations (note that only persons can witness, observe, and express).

Universal generalization has specific rules that must be followed [pp. 454 - 457]. None of the constants d, s, or c appears in a premise nor were these constants derived from existential instantiation. I did not use any of the constants in final line 18. And I did not apply the inferential rule of universal generalization within any of the conditional proofs (i.e., in an "undischarged assumption").

References:

1. Howard-Snyder F, Howard-Snyder D, Wasserman R. The Power of Logic, 4th ed. McGraw-Hill Higher Education, 2009.
2. http://www.coursesmart.com/978-0-07-731591-7?gclid=CMe_s_XB2qUCFQlubAodiBBNlw. Accessed August 23, 2011.